An Effective Strong Gravity induced by QCD

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Abstract

We show that, when quantized on a curved "intra-hadronic background", QCD induces an effective pseudo gravitational interaction with gravitational and cosmological constants in the GeV range.

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1 Introduction

In the pre-QCD phenomenology of strong interactions, several features suggested that a strong gravity [1] is at work in the hadronic world. In this phenomenology one has linearly rising Regge trajectories which suggest that hadrons be interpreted as black holes of the strong gravity (see ref [1] for a wide review). In addition to this, the success of the hadronic string picture suggests the presence of a strong gravity in its closed string sector.

More recently, the subject was revived in a series of papers by Ne'eman and Sijački who try to identify the strong gravity (which they call a pseudo-gravity) in the infrared sector of QCD [2], and in a paper by Salam and Sivaram [3], connecting strong gravity with confinement.

In the present letter, we intend to show that it is possible to derive the strong gravity as an effective theory induced by QCD when quantized in a "curved hadronic space-time" (which we call "aggravated QCD"). To do this we use the method of Adler and Zee (see section 2). This method leads to an induced gravitational constant G_{ind} and an induced cosmological constant Λ_{ind} which are in principle calculable using the parameters of the theory of matter quantized on a flat space-time. Unfortunately, for a description of the true gravity, it seems difficult with this method to obtain, for a given theory of matter, a gravitational constant compatible with G_{Newton} and a cosmological constant, which would not violently contradict astrophysical observations. When applying the Adler and Zee method to QCD quantized on this "curved hadronic space-time" it leads to a strong gravitation like interaction induced by QCD. The large induced cosmological constant, which is a difficulty for true gravity, may be an advantage for strong gravity as it acts over a finite range.

2 The method of Adler and Zee

The idea of this method (see [4] for a complete review) was to derive Einstein's gravity as an effective theory induced by the quantum fluctuations of the matter fields. The fundamental Lagrangian is thus written as

$$\mathcal{L} = \mathcal{L}_{matter}^{cov}(\Phi, g_{\mu\nu}) + \mathcal{L}_{grav}(g_{\mu\nu}) \tag{1}$$

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[1] i.e. gravitational interaction with a coupling constant G_f 10^{38} times larger than the Newton's constant and, a spin two mediator with vacuum quantum numbers identified as the f_2 meson.

where Φ represents the matter fields, $g_{\mu\nu}$ the metric field, $\mathcal{L}_{matter}^{cov}$ the "covariantized" matter Lagrangian and $\mathcal{L}_{grav}$ the gravity Lagrangian.

If we consider $\mathcal{L}_{matter}^{cov}$ renormalizable, without bare masses and $\mathcal{L}_{grav}$ purely quadratic in the curvature [2], then $\mathcal{L}$ is scale invariant. This invariance is dynamically broken by renormalization. This fact renders calculable the parameters of an effective low energy theory generated by summation over matter fields.

The effective theory is defined by:

$$\exp(i\mathcal{S}_{eff}[g_{\mu\nu}]) = \int \mathcal{D}\Phi \exp(i \int d^4x \sqrt{-det(g_{\mu\nu})}\, \mathcal{L}(\Phi, g_{\mu\nu})) \tag{2}$$

with

$$\mathcal{S}_{eff}[g_{\mu\nu}] = \int d^4x \sqrt{-det(g_{\mu\nu})}\mathcal{L}_{eff}(g_{\mu\nu}).$$

For slowly variable metrics, $\mathcal{L}_{eff}$ can be expanded in powers of $\partial_\lambda g_{\mu\nu}$ so that,

$$\mathcal{L}_{eff}(g_{\mu\nu}) = \mathcal{L}_{eff}^{(0)}(g_{\mu\nu}) + \mathcal{L}_{eff}^{(2)}(\partial_\lambda\partial_\tau g_{\mu\nu}, (\partial_\lambda g_{\mu\nu})^2) + O[(\partial_\lambda g_{\mu\nu})^4]. \tag{3}$$

It can be identified with an Einstein-Hilbert Lagrangian, up to the second order, if we write

$$\mathcal{L}_{eff}^{(0)}(g_{\mu\nu}) = \frac{1}{8\pi}\frac{\Lambda_{ind}}{G_{ind}} \quad \mathcal{L}_{eff}^{(2)}(\partial_\lambda\partial_\tau g_{\mu\nu}, (\partial_\lambda g_{\mu\nu})^2) = \frac{1}{16\pi G_{ind}}R. \tag{4}$$

where R is the scalar curvature built from $g_{\mu\nu}$. Application of $\delta/\delta g^{\rho\sigma}$ on equation (2) gives [3]

$$\frac{1}{16\pi G_{ind}}(R_{\rho\sigma} - \frac{1}{2}R\, g_{\rho\sigma} + \Lambda_{ind}g_{\rho\sigma}) = \frac{1}{2} < T_{\rho\sigma} >_{g,0} \tag{5}$$

where we have defined :

$$\frac{1}{2} < T_{\rho\sigma} >_{g,0} = \frac{1}{\sqrt{-det(g_{\mu\nu})}} \frac{\int \mathcal{D}\Phi \frac{\delta}{\delta g^{\rho\sigma}} S_{matter}^{cov} \exp(iS)}{\int \mathcal{D}\Phi \exp(iS)} \tag{6}$$

where the notation $<\;>_{g,0}$ indicates that the mean value is taken in the background $(g_{\mu\nu})$ and in the vacuum (0). We will see that equation (5) can be used under certain conditions to determine $\frac{\Lambda_{ind}}{G_{ind}}$ and $\frac{1}{16\pi G_{ind}}$. Through dynamical scale breaking in flat space-time, eq.(5) leads to [4]:

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[2]See equation (16) in Section 3 below.

[3]We follow the convention of [5] save for the metric which has here signature $(+, -, -, -)$

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$$\frac{\Lambda_{ind}}{2\pi G_{ind}} = <T^\mu_\mu>_{\eta,0} = \frac{\beta(g)}{2g}<F^a_{\mu\nu}F^{\mu\nu}_a>_{\eta,0} \tag{7}$$

$$\frac{1}{16\pi G_{ind}} = \frac{i}{96}\int d^4x x^2 (<\tau(T(x)T(0))>_{\eta,0} - <T(0)>^2_{\eta,0}) \tag{8}$$

where τ denotes the time ordered product.

We can pose the question as to wether the flat space-time approximation is valid in this case. In fact, splitting $g_{\mu\nu}$ into a background metric and a quantum fluctuation, $g_{\mu\nu} = \overline{g_{\mu\nu}} + \epsilon h_{\mu\nu}$, this approximation means $\overline{g_{\mu\nu}} = \eta_{\mu\nu}$ and ϵ small. It is justified at high energy scale, if on the one hand we do not consider the presence of background matter fields and if the classical equations for $\overline{g_{\mu\nu}}$ are solved in $\mathbf{R}^4$ without boundaries and on the other hand, if the theory is asymptotically free [4]. As $\frac{\Lambda_{ind}}{G_{ind}}$ and $\frac{1}{16\pi G_{ind}}$ are renormalisation group invariants, we can determine them for any energy scale μ and in particular in the limit of large energy scales.

According to the phenomenon of dimensional transmutation (see the so-called Gross-Neveu theorem [6]), the dimensionless coupling constant is exchanged in favor of the mass gap of the theory, $\mathcal{M}(g(\mu),\mu)$. $\mathcal{M}(g(\mu),\mu)$ is solution of the renormalization group equation,

$$[\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g}]\mathcal{M}(g,\mu) = 0 \tag{9}$$

With equation (9) one may express any physical quantity $\mathcal{P}$ of canonical dimension $d_{\mathcal{P}}$, as

$$\mathcal{P} = c\mathcal{M}^{d_{\mathcal{P}}}$$

where c is a dimensionless constant. With this and from (7) one has immediately, for a $SU(N_c)$ gauge theory,

$$\frac{\Lambda_{ind}}{2\pi G_{ind}} = -\frac{2\pi}{24}(11N_c - 2N_f)c\mathcal{M}^4 \tag{10}$$

where c is a numerical constant of the order of unity. The result for (8) is more difficult to obtain, in [7] upper and lower bounds have been derived:

$$\frac{5}{48ln(10)}(N_c^2 - 1)\mathcal{M}^2 \leq \frac{1}{16\pi G_{ind}} \leq \frac{25}{12\pi^2}(N_c^2 - 1)\mathcal{M}^2 \tag{11}$$

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[4]As is the case both for an SU(N) gauge theory and for scale invariant gravitation [9]

It is clear when we see (10) and (11) that this method cannot be applied to true gravitation at least in this form because it gives Λ_{ind} of order of $\frac{1}{G_{ind}}$. It will however, be very useful when applied to a "pseudo gravity" associated with the strong interaction.

3   QCD in a curved intra-hadronic space-time

3.1   The intra-hadronic background field

We first remark that if one starts from standard QCD as the matter input, the method of Adler and Zee leads to induced gravitational and cosmological constants with values in the GeV range since $\mathcal{M}_{QCD}(g,\mu)$ is itself in this range, and since QCD is asymptotically free. Of course such an induced gravity would have nothing to do with the true universal gravity, its metric field is certainly not the universal metric of space-time.

We argue that due to the finite range of strong interactions, it is impossible to probe with strongly interacting particles the structure of space-time inside a hadron, in such a way that there is no compelling reason to assume this space-time to be flat. On the other hand, because of confinement, the only space-time seen by the quarks and the gluons is this intra-hadronic space-time. We thus modify QCD by quantizing it on a "curved intra-hadronic background".

We treat $G_{\mu\nu}(x)$, the intra-hadronic metric field as: (i) an *independant* field, namely a field which does not depend on the matter fields, (ii) a *specific* field, which means that the photon and the weak bosons are coupled to quarks through the flat metric $\eta_{\mu\nu}$ and not through $G_{\mu\nu}(x)$.

3.2   The lagrangian of "aggravated QCD"

The Lagrangian of our theory, which we have named "aggravated QCD" can then be written as

$$\mathcal{L}_{had} = \mathcal{L}_{PG}(G_{\mu\nu}) + \mathcal{L}_{QCD}^{cov}(A_\mu^a, \psi^{i,\alpha}, G_{\mu\nu}) \tag{12}$$

where, the subscript "*had*" means that no non-hadronic field is involved, $\mathcal{L}_{PG}$ is the pseudo gravitational Lagrangian build from $G_{\mu\nu}(x)$ and $\mathcal{L}_{QCD}^{cov}$ is the standard QCD Lagrangian "covariantized" by means of $G_{\mu\nu}(x)$. In order to "covariantize" a Lagrangian involving spinors it is usefull to decompose the metric $G_{\mu\nu}(x)$ in terms

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of the tetrads e_μ^m

$$G_{\mu\nu} = e_\mu^m \, \eta_{mn} \, e_\nu^n \tag{13}$$

where the latin indices $(m, n, \ldots)$ correspond to coordinates in the tangent space (M_4) and η_{mn} is the Minkowski metric in M_4, $\eta_{mn} = \text{diag}(+1,-1,-1,-1)$. We thus write

$$\mathcal{L}_{QCD}^{cov}(A_\mu^a, \psi^{i,\alpha}, G_{\mu\nu}) = -\frac{1}{4} F_{\mu\nu}^a F_{\rho\sigma}^a \frac{1}{2} (G^{\mu\rho} G^{\nu\sigma} - G^{\mu\sigma} G^{\nu\rho}) + \bar{\psi}_{i,\alpha} \gamma^\mu \overset{\leftrightarrow}{\nabla}_\mu \psi^{i,\alpha} \tag{14}$$

where

- the gluon fields are denoted by A_μ^a, $a = 1, \ldots, N_c^2 - 1$, (N_c is the number of colors),

- the massless quark fields are denoted by $\psi_{i,\alpha}$, $i = 1, \ldots, N_c$, $\alpha = 1, \ldots, N_f$ (N_f is the number of quark flavors),

- $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + ig f_{bc}^a A_\mu^b A_\nu^c$, ($f_{bc}^a$ are the structure constants of the $SU(N_c)$ group), g is the bare QCD coupling constant,

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$$\gamma^\mu \overset{\leftrightarrow}{\nabla}_\mu = e_m^\mu [\gamma^m (\overset{\leftrightarrow}{\partial}_\mu - ig A_\mu^a \tau_a) + \frac{i}{2} \Gamma_{n\mu p} \{\gamma^m; \Sigma^{np}\}], \tag{15}$$

  where

  - $\overset{\leftrightarrow}{\partial}_\mu = \overset{\leftarrow}{\partial}_\mu - \overset{\rightarrow}{\partial}_\mu$,
  - τ_a are the generators of $SU(N_c)$,
  - $\Gamma_{n\mu p}$ are the Cristoffel symbols associated with $G_{\mu\nu}$,
  - γ^m are the Dirac matrices.
  - $\Sigma^{np} = \frac{i}{2}[\gamma^n, \gamma^p]$,

In order to preserve scale invariance, which holds if quarks are massless, the pseudo-gravitational Lagrangian is purely quadratic in the curvatures:

$$\mathcal{L}_{PG}(G_{\mu\nu}) = \alpha \, R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + \beta \, R_{\mu\nu} R^{\mu\nu} + \gamma \, R^2 \tag{16}$$

where $R^\mu_{\nu\rho\sigma}$, $R_{\mu\nu}$ and R are respectively the Riemann tensor, the Ricci tensor and the scalar curvature associated with the metric field $G_{\mu\nu}(x)$, and where α, β and γ are dimensionless coupling constants.

Such a pseudo-gravitational Lagrangian has many advantages for our purpose. (i) It ensures renormalizability [8] and (ii) asymptotic freedom [9] for the pseudo-gravitational interaction. (iii) It leads to a $\frac{1}{p^4}$ high momentum behavior for the pseudo-graviton propagator in such a way that we can expect aggravation not to modify the $\beta(g)$ function of QCD. The price to pay these good features is the possible occurence of ghosts at least at the perturbative level.

3.3 Symmetry properties of aggravated QCD

By construction, aggravation adds a symmetry to standard QCD, namely "general hadronic covariance", the invariance by a general local change of the intra-hadronic coordinates at which the quark and gluon fields are defined, whereas it preserves its symmetries. In fact since $G_{\mu\nu}$ does not depend on the QCD fields, the aggravation of QCD preserves the $SU(N_c)$ gauge invariance. This feature has to be contrasted with the breaking of gauge invariance implied by the metric used in ref. [2] and [3]. The fact that the photon and the weak bosons couple to quarks through $\eta_{\mu\nu}$ and not through $G_{\mu\nu}$ means that electromagnetic and weak interactions break general hadronic covariance, a specific symmetry of strong interactions in the same way as they break isospin invariance, another specific symmetry of strong interactions.

It turns out that chiral symmetry, a property of QCD with massless quarks, is also preserved by our aggravation procedure. To see this it is enough to note that the only term which may pose a problem, namely the fermionic term in (14), involves, as shown in (15), only odd numbers of Dirac matrices and is thus chiral invariant:

$$\bar{\psi}\gamma^\mu \overleftrightarrow{\nabla}_\mu \psi = \bar{\psi}_R \gamma^\mu \overleftrightarrow{\nabla}_\mu \psi_R + \bar{\psi}_L \gamma^\mu \overleftrightarrow{\nabla}_\mu \psi_L \qquad (17)$$

where the eigenvectors of chirality are defined by

$$\psi_{\substack{R\\L}} = \frac{1 \pm \gamma_5}{2}\psi \qquad (18)$$

As in ordinary QCD with massless quarks, chiral symmetry is thus broken only dynamically in aggravated QCD. Nevertheless, since the conservation of chiral symmetry currents must be written in terms of the covariant derivative i.e. $\partial_\mu(\sqrt{-G}J^\mu) = 0$, we can have departures from standard chiral theory. However, on the one hand, the ultra-violet decoupling of pseudo-gravity guarantees that we recover the standard formula at high energy, and on the other hand, the effective metric which we

propose below -see eq.(20)- does not contribute to $\sqrt{-G}$ [5]. In general, as in (20), we expect pseudo-gravity not to contribute at energies lower than $\mathcal{M} \equiv \Lambda_{QCD}$, and thus to preserve one of the main assets of standard QCD, namely chiral perturbation theory.

4   The induced Strong Gravity

The Lagrangian of aggravated QCD satisfies all the criteria (renormalizability, scale invariance, asymptotic freedom, absence of bare Einstein-Hilbert terms) to lead through the method of Adler and Zee to induced gravitational and cosmological constants which are finite and in principle calculable in terms of the matter theory quantized on a flat space-time. One can thus say that aggravated QCD induces an effective strong gravity. This strong gravity is characterized by its metric field $G_{\mu\nu}$, by its strong gravitational constant $G_f = G_{ind}$ and its strong cosmological constant $\Lambda_f = \Lambda_{ind}$. With $N_c = 3$ and $\mathcal{M} = \Lambda_{QCD} \sim 200 MeV$, we find that G_f and $-\Lambda_f$ [6] in the GeV range.

A first application of our scheme consists on trying to find the metric of the effective pseudo-gravitation induced by aggravated QCD. To this end one has to consider eq.(5), in which $g_{\mu\nu}$ is replaced by $G_{\mu\nu}$. Despite its appearance, this equation should be understood as a definition of an effective long distance energy-momentum tensor rather than as the Einstein equation itself. We thus can write

$$< T_{\rho\sigma} >_{G,0} \equiv T_{\rho\sigma}^{eff} = \frac{1}{8\pi G_{ind}}(R_{\rho\sigma} - \frac{1}{2}R\,G_{\rho\sigma} + \Lambda_{ind}\,G_{\rho\sigma}) \qquad (19)$$

If one wants to obtain the intra hadronic metric induced by the quantum fluctuations of the standard matter theory, one has to equal to zero the full (QCD plus pseudo-gravity) effective energy-momentum tensor, in such a way that eq.(19) can now be solved as an Einstein equation in the vacuum. This equation leads to a closed space-time since Λ_{ind} is negative. In this situation, the "non abelian Gauss theorem" implies that the only physical states are color-singlet. For instance, if the vacuum fluctuations which have been integrated on, have been created by a quark source situated at a given point of space-time, then there necessarily exists an antiquark (or a diquark) in the light cone of the quark on which the color field lines must reconverge.

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[5]except through the standard spheric term

[6]with our conventions, the induced cosmological constant is negative

The solutions of the vacuum equation involve a de Sitter intra-hadronic space-time. The metric can take several forms, including, in a comoving frame, the one of an ever expanding space-time. However it can be useful, for our purpose, to interpret the solution as it can be seen by an extra hadronic probe which couples to quarks through $\eta_{\mu\nu}$. We obtain this way the static solution of the vacuum equation

$$ds^2 = (1 + \frac{1}{3}\Lambda_{ind}r^2)dt^2 - (1 + \frac{1}{3}\Lambda_{ind}r^2)^{-1}dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2 \qquad (20)$$

This metric is flat at short distance (in accordance with asymptotic freedom) and presents a singularity at $r_s = \sqrt{\frac{-3}{\Lambda_{ind}}}$ which can be interpreted as the radius of a "gravitational bag". In this metric, the equation of an outgoing null radial geodesic is

$$\frac{t - t_0}{r_s} = \frac{1}{2}\ln\left|\frac{1 + r/r_s}{1 - r/r_s}\right| \qquad (21)$$

showing that the chromo-electric field produced by a quark at the origin never reaches $r = r_s$ and is thus confined at a distance smaller than r_s. We note that this solution is not a black hole; it is rather the exterior of the bag which resembles the interior of a Schwartchild black hole. The anti-screening effect of vacuum polarization in QCD (see [10]) is simulated in terms of a strong gravitational effect.

This encouraging result has to be considered as a first stage for a better understanding of the pseudo-gravity induced by QCD. Much more work is needed (and in progress), in particular in the study of the "fundamental" lagrangian and of the modifications of QCD implied by our aggravation procedure, and in the construction of effective theories with background quark and gluon fields. One of the first applications of aggravated QCD could be to relate its metric field $G_{\mu\nu}(x)$ to the so called "hard Pomeron" which is advocated in the interpretation of the recent Hera data [11].

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